World Journal of Environmental Sciences & Development Research
Vol. 1. No. 1, March 2015, pp. 1 - 8
Available online at http://wjesdr.com/

Research article

MATHEMATICAL MODELLING OF SHIGALLAE TRANSPORT INFLUENCED BY IMMOBILE VELOCITY IN A NATURAL POND, NIGER DELTA OF NIGERIA

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Abstract

The depositions of shigellae in natural were evaluated through applications of mathematical modeling techniques. This study were carried out to determine the behaviour of shigellae in natural pond from point sources of discharge, various investigation were carried out that generated numerous information on the sources of these contaminants, these evaluation generated several causes of variation on the depositions of shigellae inside the pond, mathematical modeling approach were another sources of application that monitor the deposition growth rate and degradation of shigellae in natural pond, the expressed system generated the governing equation, these produced the derived model that will monitor the transport of the microbes at various condition, experts will find the developed model useful in monitoring and calibrating of shigellae depositions and it rates of concentration in natural pond.

Keywords: modeling, shigellae, immobile velocity and natural pond.

1. Introduction

The challenges facing water quality modellers should be contemplated in the wider perspective of risk-based decision support. Firstly, a high degree of model uncertainty is not necessarily an undesirable outcome, and undoubtedly is preferable to no indication of reliability at all. Secondly, uncertainty in environmental models should be viewed as a source of risk, as is traditional in other fields of engineering (e.g. Tung 1996 Manoj et al 2006), and should be used to establish and achieve an acceptable failure probability in terms of water quality status, rather than be used to decry the modelling approach (Beven 2000a,1992,). Given that risk is a concept that can be used to integrate external criteria such as economics and safety, as well as integrating the model result over the relevant model responses, expressing results as risk is potentially attractive and seems inevitable. Thirdly, it is worth noting that, in the context of decision-support, we are not justified in investing resources in modelling (including the identification of prediction uncertainty) unless this will be instrumental to the decisions that need to be made (Beven 1993, 1992 and 1998). Therefore, we should keep sight of the modelling task, and accept that (very) approximate solutions may be appropriate (Manoj et al 2006).

Uncertainty in a water quality simulation model is inevitable due to the difficulty of identifying a single model (including grid-scale, process formulations and parameter values) which can accurately represent the water quality under all required model tasks (see the discussions of Beck 1987, Van Straten 1998, and Adams and Reckhow 2001).

Therefore, models identified for one case study cannot be used with any confidence for another. Literature which describes established formulations and parameter values (Bowie et al. 1985, Thomann and Mueller 1987, Chapra 1997) is evidence of the wide range of models which are equally justified prior to observing a system's behaviour in detail, and that the uncertainty associated with modelling water quality on the basis of prior knowledge is extremely large. Given that it is desirable to evaluate the performance of models with respect to observed water quality data, the accuracy, frequency and relevance of the available data dictates the attainable degree of certainty in the model. Unfortunately, water quality data can be expensive to collect and analyse, often requiring special handling and analysis in laboratories. This means that data to support model identification are generally sparse, often coming from sampling programmes which are fixed in frequency and location for regulation purposes, rather than designed to encapture the system's dynamic responses as required for successful model identification (Berthouex and Brown 1994).

Excess nitrogen, phosphorus, and sediment loadings have resulted in water quality degradation within the Upper Mississippi River and its tributaries. This is particularly true for watersheds draining in portions of Iowa, which are generally greatly impacted by agricultural nonpoint source pollution. Kalkoff et al. 2000) report that nitrogen and phosphorus levels measured in several large eastern Iowa watersheds, which drain to the Mississippi River, were among the highest found in the Corn Belt region and in the entire United States as part of the U.S. Geological Survey (USGS) National Water- Quality Assessment Program. Schilling and Libra (2000) state that annual export of nitrate from surface waters in Iowa was estimated to be about 25% of the nitrate that the Mississippi River delivers to the Gulf of Mexico, despite Iowa occupying less than 5% of its drainage area. The nitrate load discharged from the mouth of the Mississippi River has been implicated as the primary cause of the seasonal oxygen-depleted hypoxic zone that occurs in the Gulf of Mexico, which has covered upwards of 20,000 km2 in recent years (Rabalais et al., 2002).

Several studies have been performed in the RWW to quantify nitrate concentration patterns and corresponding stream flow relationships. Schilling and Lutz (2004) examined a 28-year record (1972- 2000) of stream flow and nitrate concentrations measured in the Raccoon River and reported evidence of strong seasonal patterns in annual nitrate concentrations, with higher concentrations occurring in the spring and fall. No long-term trends in nitrate concentrations were noted during the entire period. Schilling and Zhang (2004) described nitrate loading patterns in the Raccoon River and found that nitrate losses in base flow comprised nearly two-thirds of the total nitrate load over the same 28-year monitoring period. They also found that seasonal patterns of nitrate loads were similar to nitrate concentration patterns, with base flow contributions to nitrate loads greatest in the spring and after fall, when base flow contributed more than 80% of the total nitrate export.

2. Theoretical Background

The behaviour contaminants in pond are base on several factors in such a system, such deposition of microbe's shigellae in natural pond from point source of discharge were observed to deposit through drains collecting biological waste in nearby communities, the system of natural pond were found predominantly depositing different parameters that generate various accumulation of waste in the system, base on these conditions the concentration of shigellae were investigated to predominantly deposit in the natural pond. These conditions expressed the influences from the system such as the deposition of immobile velocity or wind velocity that cannot influences flow in the pond, the system at this condition can also be identified as batch system. The condition influences the exponential rate of the microbes inside the pond. Other parameters that should influences the system reflect their behaviour on the rate of shigella deposition, these are base on the rate of their depositions, including dispersion influences of these pollutant, this will definitely affect the aquatic lives faster base on the rate of flow, this also pressure increase in concentration whereby regenerating of oxygen demand in the pond will experiences faster reactions. The study expresses some conditions that pressure the exponential rate of shigella in natural pond, in some condition it occur that the microbes become substrate to other aquatic lives in the pond, but several negative impact will always reflect on the aquatic lives in the pond, base on these conditions it becomes imperatives that the extend of deposition and rate of contamination through point sources discharge should be investigated, the derived mathematical expression establish various conditions that has pressured the system including other rate of shigellae degradation in the pond.

3. Governing Equation

$$K\frac{\partial c}{\partial t} = Dx\frac{\partial^2 c}{\partial x^2} + Dy\frac{\partial^2 c}{\partial y^2} - U\frac{\partial c}{\partial x} - V\frac{\partial c}{\partial y} + Kc \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

The governing equation expressed here were developed to monitor the behaviour of shigellae in a natural pond, the developed governing equation will definitely express the effect of those parameter that make the system to analyzed various ways, that these variables will pressure the deposition of shigellae in natural pond. several way in which this parameters influences the rate of shigellae deposition will be evaluated in the study base on the derived solution from the governing equation.

$$\lambda^2 = Dx\frac{\partial^2 c}{\partial x} - U\frac{\partial c}{\partial x} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

$$\lambda^2 = Dy\frac{\partial^2 c}{\partial y^2} - V\frac{\partial c}{\partial y} + Kc \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\left.\frac{\partial c}{\partial x}\right|_{x=L} = 0 ;\ C(o^-) = C(o^+) = Co,\ x = o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Where *L* is the distance in *x*-direction of the pond

$$\left.\frac{\partial c}{\partial y}\right|_{y=o,\ H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Where H is the depth of the pond,

The expression to monitor the behaviour of the microbes in several conditions should be established, the rate of deposition under some parameters that express their rate of accumulation are determined through the integration of some boundary values. This concept are applied to monitor the limit of reaction on the deposition of shigellae within the pond, these are base the level it expression in the derived solution through its integrations, such expression reflect the behaviour of its rates of concentrations as its is express in the natural pond .

The solution for equation (1) is of the form:

$$C(x,y) = C(x) + C(y) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

We consider equation (2), using Bernoulli's method of separation of variations: -

$$\lambda^2 = Dx\frac{\partial^2 c}{\partial x^2} - U\frac{\partial c}{\partial x}$$

$$\lambda^2 = Dx\, X^{11} - UX^1$$

$$DX^{11} + UX^1 - \lambda^2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$X = A_2\, \ell^{m_1 x} + A_3\, \ell^{-m_2 x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The expressions in these dimensions establish the behaviour of shigellae under the influences of various parameters that pressure the rate of concentration within the pond, separating the parameters applied in other to determine various variable influences including their rates of exponential and degradation condition within the pond, it is noted that these parameters that made off these system should be descretize through these application, these expression determine various rate of pressure reflecting on the level of shigellae deposition in the system.

Where $M_1 = \frac{U + \sqrt{U^2 + 4Dx\lambda^2}}{2Dx}$ *and* $M_2 = \frac{U - \sqrt{U^2 + 4Dx\lambda^2}}{2Dx}$

This can be expressed in this form

$$C(x) = A_1 \ell^{m_1 x} + A_2 \ell^{-m_2 x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Subject (9) to equation (4), we have

$$C(x,t) = \frac{CoM_2 \ell^{M_2 L}}{M_2 \ell^{M_2 L} + M_1 \ell^{M_1 L}} \left[\ell^{M_1 x} + \frac{M_1 \ell^{M_1 L}}{M_2 \ell^{M_2 L}} \ell^{-M_2 x} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

The expressions monitor the exponential behaviour of the microbes deposited within the pond, the derived solution monitor the rate of shigellae increasing through deposited mineral in the pond, the derived solution monitor the system under the influences on microelement that may deposit through any sources thus increase the growth rate of shigellae inside the pond, the established condition in [10] monitor the growth rate through these applications. The growth rate of shigellae without any inhibitors may experiences accumulation, through the deposition of immobile velocity in natural pond.

Similarly, equation (3) can be solved as this

$$\lambda^2 = Dy \frac{\partial^2 c}{\partial y^2} - V \frac{\partial c}{\partial y} + Kc \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\lambda^2 = Dy\, Y^{11} + VY^1 + Kc \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\Rightarrow Dy\, Y^{11} - VY^1 + Kc - \lambda^2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$Y = B_1\, Cos\, n_1 Y + B_2\, Sin\, n_2 y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Where $n_1 = \dfrac{V + \sqrt{V^2 - 4Dy(K - \lambda^2)}}{2Dy}$ and $n_2 = \dfrac{V - \sqrt{V^2 - 4Dy(K - \lambda^2)}}{2Dy}$

The expressions here monitor the rate of dispersion within the pond, the rate of shigellae intrusion through point sources discharge into the pond were considered on the derived solution, the influences from the immobile velocity were observed to pressure the system on the rate of spread within the pond, the deposition of shigellae experienced the influences from the rate of immobile velocity including their various reaction within other aquatic lives inside the pond.

So that the expression can be written as:

$$C(y) = \left(B_1\, Cos\, n_1 y + B_2\, Sin\, n_2 y \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Subject equation (14) to equation (5), yield:

$$C(y,x) = \frac{CoM_2 \ell^{M_2 L}}{M_2 \ell^{M_2 L} + M_1 \ell^{M_1 L}} \left[bn\, Cos \frac{n\pi}{H} y \right] n, 1, 2, 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

Summary over Fourier series, in the section, we have

$$C(y,x) = \frac{Co\,M_2\ell^{M_2L}}{M_2\ell^{M_2L} + M_1\ell^{M_1L}}\left[\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n\, Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (16)$$

$$\Rightarrow \frac{a_o}{2} = \frac{1}{H}\int_o^H f(u)\,du,\; b_n = \frac{2}{H}\int_o^H f(u)\,Cos\,udu$$

Hence, $C(y,x) = \frac{1}{H}\int_o^H f(u)\,du + \frac{2}{H}\sum Cos\frac{n\pi}{H}y\int_o^H f(u)\;Cos\,udu$ ……… (17)

Hence $bo = \frac{a_o}{2}$ and $b_n = a_n$, for $n \geq 1$.

At this point, the pollutant is following with homogeneous underground velocity downward (i.e. transverse dispersions).

Substituting equation (10) and (17) into equation (6), so that we can express it in this form.

$$C(x,y) = \frac{Co\,M_2\ell^{M_2L}}{M_2\ell^{M_2L} + M_1\ell^{M_1L}}\left[\ell^{M_1x} + \frac{M_1\ell^{M_1L}}{M_2\ell^{M_2L}}\ell^{-M_2x} + bn\;Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots \qquad (18)$$

As $x = \infty,\ y = 0,\ c \to 0$ so that we can have the above expression in this form

$$C(x,y) = \frac{Co\,M_2\ell^{M_2L}}{M_2\ell^{M_2L} + M_1\ell^{M_1L}}\left[\ell^{M_1x} + bo + \sum_{n=1}^{\infty} bn\;Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots \qquad (19)$$

Where $M_1 = \frac{U + \sqrt{U^2 - 4Dx}}{Dx}$, $M_2 = \frac{U - \sqrt{U^2 + 4Dx}}{Dx}$ and $n_1 = \frac{V + \sqrt{V^2 - 4Dy\,Kc}}{2Dy}$ …. 20

The expression in [20] is the final derived model for the study, these are base on the investigation carried out in the pond, and the developed governing equation observed lots of environmental conditions that generated lots of accumulation, the developed model examined these parameters in the system at various boundary conditions, the system developed the governing equation base on the examined condition from the pond, several conditions on the rate of depositions including growth rates were observed, the rate of immobile velocity of flow on the deposition were also examined.

4. Conclusion

The behaviour shigellae in natural pond has been expressed in various dimensions, the point source discharge to the pond were observed to be the sources the pond experiences contamination, it has been observed that high rate of these contaminants intruding inside the pond are base on the point source, these condition were evaluated in various dimension under the influences of immobile velocity and other constituent inside the pond, the express developed governing equation from the formulated system thoroughly assess some possible conditions that may increase or degrade the contaminant in the pond. The derived solution express various condition that are very possible for exponential phase of the microbes, these expressions generated the final model for the study. There is no doubt that the express model will definitely monitor the rate shigellae concentration pressured by immobile velocity in natural pond.

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